Scaling theory of quantum breakdown in solids

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We propose a notable scaling theory for general quantum breakdown phenomena. We show, taking Landau-Zener-type breakdown as a particular example that the breakdown phenomena can be viewed as a quantum phase transition for which the scaling Ansatz is developed. Its application to Zener-type breakdown in Anderson insulators and quantum quenching has been discussed.

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The behavior of solid-state electronic systems in nonequilibrium conditions, particularly the insulator-metal transitions leading to dielectric breakdown in constant electric field just beyond a particular threshold, has been a matter of considerable investigation. This started with Zener,[1,2] considering the charge-excited carrier-generation process as tunneling through the band gap Δ, from ground state to excited states in one-dimensional band insulator, using the time-independent as well as time-dependent gauge.[2,3] Recent years have noticed increasing interests in the nonlinear phenomena of quantum breakdown in many-body systems put in strong dc electric fields, in intense laser fields, etc. The interest also comes because of the fact that the high sensitivity (drastic changes) of electronic states to external field near the phase boundary finds its application in dielectric breakdown based current-switching devices.[4] However, more interesting is its relevance to nonequilibrium statistical physics where one can observe the behavior of uncorrelated as well as strongly correlated electronic system near phase boundary under nonequilibrium conditions and in the study of quantum-defect production in rapid quench of quantum fluctuations (see, e.g., Das and Chakrabarti[5]).

The tunneling rate P of charge-excited carriers through a gap Δ in dc electric field E (for the case of band insulator) has been shown to be expressed in closed form, which is proportional to $\exp[-(\mathcal{E}_{th}/Ee)]$ with $\mathcal{E}_{th}\sim\Delta^2$ and e denoting the carrier charge.[1–3] Recently, it has been shown that such Landau-Zener transition estimate can be straightforwardly extended to Mott insulators as well with the same exponent value,[6,7] though the carriers are different. In band-insulator holes and electrons undergo tunneling through band gap whereas in Mott insulator doublons and holes carry out the same role through Mott gap.

Within the present framework, it seems to be virtually impossible to obtain such an exact expression for the tunneling rate of the carriers in a disordered quantum systems such as the Anderson insulators.[8] Our purpose here is to develop a simple scaling theory for such quantum breakdown phenomenon and explore the Landau-Zener-type[3] breakdown in, e.g., Anderson insulators (say, in three dimension). It may be mentioned that the growth of fluctuation correlations near various well-studied classical breakdown points have already helped establishing a phase-transition picture, leading to well-established scaling theories for phenomena, such as fracture, etc. (see, e.g., Chakrabarti and Benguigui[9]). Also, precise critical behavior of breaking in, e.g., the fiber bundle such as models of fracture is now quite established.[10]

From elementary quantum mechanics (see e.g., Merzbacher[11]), one gets the tunneling rate through an energy barrier of (large) width w to be proportional to $\exp(-c\kappa w)$, where κ is the damping factor or "imaginary momentum/wave number" determined by the height of the barrier and c is some dimensionless constant. Considering the quantum breakdown phenomena as quantum phase transitions (see e.g., Sachdev[12]), we assume that near the breakdown point, the macroscopic correlation length ξ will provide the only scale governing the behavior of physical quantities of the system. We assume this to be true for the tunneling rate P as well. Hence, the width w is scaled by ξ and "momentum," given by the inverse of the length scale, is considered to be proportional to inverse of ξ. Hence, the dimensionless tunneling-rate expression takes the form

$$P \sim \exp\left[-c\left(\frac{a}{\xi}\right)\left(\frac{w}{\xi}\right)\right]. \quad (1)$$

Here, a denotes lattice constant, the only other (microscopic) length scale in the system. The above gives our proposed scaling form for tunneling probability across a barrier of width w near a quantum phase transition characterized by the correlation length ξ determined by the barrier height.

We now proceed with the application of the above scaling Ansatz. In presence of a constant dc field, the bands effectively get tilted in field direction, which causes each energy band degenerate with the others. Therefore, electron can pass from lower energy band to upper one traversing an effective distance w in the direction of field and we envisage the process involved as tunneling. The effective width w can be estimated assuming that the energy Eew acquired from the field E over the width w will compare with the gap Δ, giving $w=\Delta/Ee$. We assume, the correlation length diverges as $\Delta^{-\nu}$ at criticality ($\Delta=0$) with exponent[12] ν and additionally we assume that even up to the breakdown point the external field does not affect the correlation-length exponent. Therefore the tunneling-rate expression becomes

$$P \sim \exp\left[-\left(\frac{ca}{Ee}\right)\Delta^{\gamma}\right]; \quad \gamma = 1 + 2\nu. \quad (2)$$

For the band and Mott insulators, putting ν as 1/2 (see, e.g., Imada *et al.*[13]), we get $P\sim\exp(-\mathcal{E}_{th}/Ee)$, $\mathcal{E}_{th}\sim\Delta^2$, the same expression as was obtained earlier.[2,3,6,7] For Mott insulators, the recent study by Oka *et al.*[6,7] supports the above expression as well, where the gap is determined by the electronic correlation.

Since the system response near breakdown is governed essentially by the correlation-length exponent, one can make some estimate for the electrical response in quantum systems with disorder, equipped with the knowledge of its correlation length around criticality in such disordered systems. Studies on Anderson transition show that the electron as a quantum particle cannot diffuse through the geometrically percolating path due to the coherent backscattering (of the wave function) from the random geometry of the clusters in dimensions less than three.[8] Since all the states on any such percolating lattice gets localized (exponentially), electrons do not diffuse through the disordered (classically percolating) lattice. In three dimension (for concentrations above the quantum percolation threshold), if the Fermi level ϵ_f lies below the mobility edge ϵ_c ($\neq 0$ for three dimension), the system remains to be quantum mechanically nonpercolating with the exponentially localized electronic states while for $\epsilon_f > \epsilon_c$ the states are extended. Hence if $\epsilon_f < \epsilon_c$, the system is an insulator and for $\epsilon_f > \epsilon_c$ it is a conductor. The scaling property in this conducting region is well established; in particular, the localization or correlation length $\xi \sim |\epsilon_f - \epsilon_c|^{-\nu}$ and precise estimates (both theoretical and experimental) of ϵ_c and ν are now available (see, e.g., Lee and Ramakrishnan[8] and Belitz and Kirkpatrick[14]). For $\epsilon_f < \epsilon_c$, one can think of an insulator-metal transition in such a noninteracting random system, analogous to quantum breakdown in band or Mott insulators, induced by (strong) dc electric fields. The critical behavior can be easily predicted utilizing the precise knowledge of localization-length exponent ν. With[8] $\nu \simeq 1$ for Anderson insulators in three dimension, the tunneling probability across the mobility gap $\Delta \equiv |\epsilon_c - \epsilon_f|$ near breakdown point, will be given by $P \sim \exp(-\mathcal{E}_{th}/Ee)$, where $\mathcal{E}_{th} \sim \Delta^{\gamma}$; $\gamma \simeq 3$. This indicates that ln P would scale as Δ^3 for Zener-type breakdown in Anderson insulators, instead of as Δ^2 for similar breakdown in band insulators.

In the context of adiabatic quantum computations, where one exploits the tunneling probability through the energy or cost barriers for local minima to reach a global minimum (may be degenerate, as in spin glass), one needs to estimate the density of defects remaining over the ground state as one quenches from a highly excited (para) state.[5] For example, in a transverse Ising glass model represented by the Hamiltonian[5]

$$H = -\sum_{(i,j)}^{N} J_{ij}\sigma_i^z\sigma_j^z - \Gamma(t)\sum \sigma_i^x \tag{3}$$

with random exchanges J_{ij} between the Pauli spins $\vec{\sigma}$, one can vary the quantum fluctuation slowly but linearly $[\Gamma(t) = 1-(t/\tau);\ 0 \leq t \leq \tau]$ to arrive at a ground state of the classical spin glass (for $\Gamma=0$ at $t=\tau$) from a disordered or a para phase (for $\Gamma=1$ at $t=0$). The amount of defect over the true ground state (giving the solution of a computationally hard problem) depends on the quenching time τ and the advantage of such annealing or quenching (tunneling) through macroscopically high or $O(N)$ barriers (instead of thermal hopping over the barriers as in classical simulated annealing[15]) might be lost if defect concentration remains high. Identifying the force Ee in Eq. (2) as the rate R of change in the energy of the system, one can easily estimate the density of defects by calculating the tunneling probability as

$$P \sim \exp\left(-\frac{\Delta^{\gamma}}{R}\right) \sim \exp\left(-\frac{\tau}{\tau_0}\right), \tag{4}$$

where $R \sim \frac{\partial}{\partial t}\langle H(t)\rangle$, the characteristic tunneling time $\tau_0 \sim \Delta^{-\gamma}$ and exponent $\gamma = 1+2\nu$. Assuming that the residual energy E_{res} (over the ground-state energy) of the quenched state to be proportional to $\int \Delta P \rho(\Delta) d\Delta$ and averaging over the Gaussian distribution[16] $\rho(\Delta) \sim \exp(-\Delta^2)$ of the local gap parameter Δ or of the local fields[5] ($h_{loc}; \Delta = \sqrt{h_{loc}^2 + \Gamma^2}$), one gets the decay rate of average residual energy as $E_{res} \sim \tau^{-4/3}$ (for large τ values) in a long-range Gaussian spin-glass model (having[17] $\nu = 1/4$). This, in fact, compares quite well with the numerical estimate of E_{res} for fairly larger Gaussian spin-glass systems studied in Matsuda *et al.*,[18] where E_{res} decays much slower than τ^{-2} (observed for smaller systems).

In brief, we propose here a generalized scaling form in Eq. (1) for the single-particle tunneling probability across a barrier, where we subsequently replace the single-particle correlation by the many-body one appropriate for a quantum many-body phase transition, obtaining thereby the generalized Landau-Zener breakdown probability. This scaling Ansatz has then been utilized for a Zener-type breakdown in Anderson insulators in three dimension. Finally, we apply it to estimate the quenching-rate dependence of the residual energy of a long-range quantum Ising spin glass when its transverse field is swept through the critical point. Comparison with numerical results seems encouraging.

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